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General dilatonic gravity with an asymptotically free gravitational coupling constant near two dimensions

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Abstract

We study a renormalizable, general theory of dilatonic gravity (with a kinetic-like term for the dilaton) interacting with scalar matter near two dimensions. The one-loop effective action and the beta functions for this general theory are written down. It is proven that the theory possesses a non-trivial ultraviolet fixed point which yields an asymptotically free gravitational coupling constant (at $\epsilon \to 0$) in this regime. Moreover, at the fixed point the theory can be cast under the form of a string-inspired model with free scalar matter. The renormalization of the Jackiw-Teitelboim model and of lineal gravity in $2+\epsilon$ dimensions is also discussed. We show that these two theories are distinguished at the quantum level. Finally, fermion-dilatonic gravity near two dimensions is considered.

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1 Introduction.

As is well known, quantum gravity (QG) based on the four-dimensional Einstein action is not renormalizable [1] (for a detailed account of the operator formalism in Einstenian gravity, see [2]). Among the different proposals for the construction of a consistent theory of QG, the $2+\epsilon$ approach offers a quite interesting way to solve this problem of nonrenormalizability of Einstein gravity. In fact, it was shown some time ago [3] that the gravitational coupling constant G in $2+\epsilon$ dimensions exhibits an asymptotically free behavior. But, unfortunately, it has been also shown that Einstein gravity in $2+\epsilon$ dimensions (a theory that has no smooth $\epsilon \to 0$ limit) is not renormalizable near two dimensions [4]. This renders the issue of asymptotic freedom of Einstein's theory near two dimensions rather hopeless.

On the other hand, there has been recently a lot of activity about two-dimensional quantum dilatonic gravity (see, for example, [5]-[10], and for reviews and more complete lists of references [11, 12]). Dilatonic gravities appear mainly as string-inspired models and it is common belief that they can provide very nice toy models for a realistic theory of QG and a reliable description of two-dimensional black holes.

Dilatonic gravity of a general form is a renormalizable theory [10, 13]. Unlike Einstein gravity, it has a smooth $\epsilon \to 0$ limit. As it was shown recently [14] —using as example dilatonic gravity without a kinetic term for the dilaton and without dilatonic potential— there exists a regime of the theory where the gravitational coupling constant is asymptotically free and the dilatonic coupling functions possess a non-trivial ultraviolet fixed point. As was seen in Ref. [14], at the fixed point the theory can be transformed to the CGHS form [5].

In the present paper we study the general theory of dilatonic gravity —which includes in the classical action a kinetic-like term for the dilaton and a dilatonic potential— near two dimensions. In the next section we start from the classical action and obtain the one-loop effective action in the covariant gauge. The renormalization group β-functions for the gravitational constant G and for the dilatonic couplings are constructed to first order in G. With the linear *Ansatz* for the dilatonic couplings and in the regime of asymptotic freedom for G the fixed points of the RG equations are found. It is shown that, at the non-trivial ultraviolet fixed point, the theory can be presented as string-inspired dilatonic gravity (not in the CGHS form) with free scalars. In Sect. 3 a similar study is carried out for fermion-dilatonic gravity. In section 4 the one-loop renormalization of two particular models —namely the Jackiw-Teitelboim one and lineal gravity— are discussed near two dimensions. It is shown that, in a Landau type gauge, lineal gravity is one-loop renormalizable but only in exactly two dimensions while, on the contrary, the Jackiw-Teitelboim model is not one-loop renormalizable, neither in exactly two nor near two dimensions.

2 General dilatonic gravity in $2+\epsilon$ dimensions.

We shall consider the theory of dilatonic gravity characterized by the following Lagrangian [15, 13]

$$L = \frac{\mu^\epsilon}{2} e^{-2Z(\phi)} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{\mu^\epsilon}{16\pi G} e^{-2\phi} R + \mu^\epsilon m^2 e^{-V(\phi)} - \frac{1}{2} e^{-2\Phi(\phi)} g^{\mu\nu} \partial_\mu \chi_i \partial_\nu \chi^i, \qquad (1)$$

where μ is a mass parameter that keeps the correct dimensions in $(2+\epsilon)$-dimensional space, G is the gravitational coupling, $g_{\mu\nu}$ the $(2+\epsilon)$-dimensional metric, R the corresponding curvature, χ_i are scalars $(i = 1, 2, \ldots, n)$, and where the dilatonic functions $Z(\phi)$, $V(\phi)$ and $\Phi(\phi)$ are supposed to be smooth enough and chosen so that $Z(0) = V(0) = \Phi(0) = 0$. The mass term m^2 is introduced in (1) in order to keep $V(\phi)$ dimensionless. Notice that an arbitrary dilatonic coupling function in front of R in (1) can be always writen (without loss of generality) under the form chosen here, by a simple redefinition of the dilaton. Our aim in this paper will be to study in detail the renormalization structure of (1) and, in particular, its renormalization group near two dimensions. A very interesting observation is the fact that, unlike $(2+\epsilon)$-dimensional Einstein theory, the theory (1) is renormalizable. Moreover, it has a smooth limit for $\epsilon \to 0$. This property allows for the possibility to study the behavior of (1) in $2+\epsilon$ dimensions by simply using the counterterms calculated already in two dimensions —in close analogy with quantum field theory in frames of the ϵ-expansion technique (for a review see [16] and [17]). It is interesting to note that, in the limit $\epsilon \to 0$, the theory [15] can be discussed in a string effective action manner [18, 19].

Studies of dilatonic gravity near two dimensions have been also carried out in Refs. [14, 20]. However, we should point out that in these works the first term in (1) has been chosen to be zero. The motivation for such choice was the fact that by a conformal transformation of the metric in (1) one can eliminate the kinetic term for the dilaton and hence the two theories appear to be equivalent. However, this is only a classical equivalence, that can be easily lost at the quantum level, as is well known. In consequence, we prefer to discuss the Lagrangian (1) that yields the most general action for a renormalizable theory of dilatonic gravity.

We choose the covariant gauge-fixing action in the form [10]

$$S_{gf} = -\frac{\mu^\epsilon}{32\pi G} \int d^d x \sqrt{-g}\, g_{\mu\nu} \chi^\mu \chi^\nu e^{-2\phi}, \qquad (2)$$

where

$$\chi^\mu = \nabla_\nu \bar{h}^{\mu\nu} + 2\nabla^\mu \varphi \qquad (3)$$

and $\bar{h}^{\mu\nu}$ is a traceless quantum gravitational field and φ a quantum scalar field, in the background field method (see [21] for an introduction).

In this gauge (2), the calculation of the one-loop effective action can be done in exactly two dimensions, with the following result (we shall drop the details of the evaluation, since

these techniques are quite well known by now, see [15, 10, 22, 23])

$$\Gamma_{div} = \frac{1}{4\pi\epsilon}\int d^dx\,\sqrt{-g}\,\Big\{\frac{24-n}{6}R + 16\pi G m^2 e^{2\phi - V(\phi)}[2+V'(\phi)] - \Big[8 - n\Phi'(\phi)^2 + 16\pi G e^{2\phi-2Z(\phi)}(2-Z'(\phi))\Big]\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\Big\}\,, \tag{4}$$

where $\epsilon = d-2$. Thus we have got the one-loop effective action, in the gauge (2), in $2+\epsilon$ dimensions. For $e^{-2Z(\phi)} = 0$ and $\Phi(\phi) =$ const. the one-loop effective action (4) coincides with the one obtained in Refs. [10, 22] in the same gauge, and for the action (1) it coincides with the result obtained in Ref. [15]. The gauge dependence of (4) may be studied in a way similar to that of the last Ref. [10].

The corresponding counterterms can be written as

$$\Gamma_{count} = -\mu^\epsilon \int d^dx\,\sqrt{-g}\,\Big[RA_1 + g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\bar{A}_2(\phi) + m^2A_3(\phi)\Big]\,, \tag{5}$$

where

$$\begin{aligned}
A_1 &= \frac{24-n}{24\pi\epsilon},\\
\bar{A}_2(\phi) &= \frac{1}{4\pi\epsilon}\Big[-8 + n\Phi'(\phi)^2 - 16\pi G e^{2\phi-2Z(\phi)}(2-Z'(\phi))\Big] \equiv A_2(\phi) + e^{-2Z(\phi)}A(\phi),\\
A_3(\phi) &= \frac{4G}{\epsilon}[2+V'(\phi)]e^{2\phi-V(\phi)} \equiv e^{-V(\phi)}\tilde{A}_3(\phi),
\end{aligned} \tag{6}$$

where the term with $Z(\phi)$ is specified separately.

We can now start with the study of renormalization. It can be done in close analogy with the approach developed in Refs. [14, 20], so no details will be given. Observe that the renormalization of the kinetic term for the dilaton, i.e. $A_2(\phi)$, originated from the renormalization of the metric, as in Refs. [14, 20], and from the contribution from $Z(\phi)$ itself (which is of the order of G). Hence, the only natural way to renormalize the theory is to consider the renormalization of Z in a homogeneous way (via Z itself). Then, in the absence of Z in (1) from the very beginning we are back to the situation of Refs. [14, 20]. With these remarks the beta-functions for the dilatonic coupling functions can be easily obtained.

A careful calculation of the dilatonic beta-functions gives the following result (see also [14, 20] for a discussion of the RG in the absence of the dilatonic kinetic term in (1), i.e. when $e^{-2Z} = 0$)

$$\beta_G = \mu\frac{\partial G}{\partial\mu} = \epsilon G - 16\pi\epsilon A_1 G^2, \tag{7}$$

$$\beta_\Phi(\phi_0) = \mu\frac{\partial\Phi(\phi_0)}{\partial\mu} = 8\pi\epsilon A_1 G(e^{2\phi_0}-1)\Phi'(\phi_0) + \frac{2\pi\epsilon^2 G}{\epsilon+1}[\Phi'(\phi_0)-1]\int_0^{\phi_0} d\phi'\, e^{2\phi'}A_2(\phi'), \tag{8}$$

$$
\begin{aligned}
\beta_V(\phi_0) &= \mu\frac{\partial V(\phi_0)}{\partial\mu} = 8\pi\epsilon A_1 G(e^{2\phi_0}-1)V'(\phi_0) \\
&+\frac{2\pi\epsilon G}{\epsilon+1}[\epsilon V'(\phi_0)-2(2+\epsilon)]\int_0^{\phi_0} d\phi'\, e^{2\phi'}A_2(\phi') - \epsilon[\widetilde{A}_3(\phi_0)-\widetilde{A}_3(0)], \qquad (9)\\
\beta_Z(\phi_0) &= \mu\frac{\partial Z(\phi_0)}{\partial\mu} = 8\pi\epsilon A_1 G(e^{2\phi_0}-1)[Z'(\phi_0)-2] - \epsilon[A(\phi_0)-A(0)] \\
&-\frac{2\pi\epsilon^2 G}{\epsilon+1}[e^{2\phi_0}A_2(\phi_0)-A_2(0)] + \frac{2\pi\epsilon^2 G}{\epsilon+1}[Z'(\phi_0)-1]\int_0^{\phi_0} d\phi'\, e^{2\phi'}A_2(\phi'). \qquad (10)
\end{aligned}
$$

Here ϕ_0 denotes the bare (non-renormalized) dilaton. We calculate the dilatonic β-functions at ϕ_0 because we are mainly interested in the functional dilatonic dependence of these β-functions. For consistency, in expressions (7)–(10) terms with $\widetilde{A}_3(0)$ and $A_2(0)$ appear. As has been mentioned in [14, 20] these terms are actually connected with the renormalization of the zero modes of the dilatonic functions. The term with $\widetilde{A}_3(0)$ in (9) appears as a result of the renormalization of the (non-essential for us) coupling constant m^2 [20]. At the same time, the terms that would carry $A(0)$ and $A_2(0)$ in (10) disappear through renormalization of the trace of the metric, where they are of next-to-leading order. (Recall that the renormalization of the trace of the gravitational field at leading order is, according to (6), an $\mathcal{O}(1)$ term.)

Let us now search for fixed points of the beta functions. The gravitational coupling constant shows asymptotic freedom, with the corresponding ultraviolet fixed point being

$$
G^* = \frac{3\epsilon}{2(24-n)}, \qquad (11)
$$

where $n < 24$ and $\epsilon > 0$. Hence, asymptotic freedom for G is obtained only when the matter central charge satisfies $0 < n < 24$ and $\epsilon \to 0$. When searching for the fixed points of the dilatonic couplings through the following *Ansatz*

$$
\Phi(\phi) = \lambda\phi, \qquad V(\phi) = \lambda_V\phi, \qquad Z(\phi) = \lambda_Z\phi, \qquad (12)
$$

we can write the beta functions under the form:

$$
\begin{aligned}
\beta_\Phi &= G(e^{2\phi}-1)\left[\frac{24-n}{3}\lambda + \frac{\epsilon}{4(1+\epsilon)}(\lambda-1)(n\lambda^2-8)\right], \\
\beta_V &= G(e^{2\phi}-1)\left[\frac{24-n}{3}\lambda_V + \frac{n\lambda^2-8}{4(1+\epsilon)}(\epsilon\lambda_V-4-2\epsilon)-4\lambda_V-8\right], \\
\beta_Z &= G(e^{2\phi}-1)\left[\frac{12-n}{3}(\lambda_Z-2) + \frac{\epsilon(n\lambda^2-8)}{4(1+\epsilon)}(\lambda_Z-3)\right]. \qquad (13)
\end{aligned}
$$

A detailed study of these equations (13) yields the following results. For the solution of Ref. [14]

$$
\lambda^* = -\frac{6\epsilon}{24-n} + \mathcal{O}(\epsilon^2), \qquad (14)
$$

one has

$$
\lambda_V^* = \frac{12\epsilon}{12-n} + \mathcal{O}(\epsilon^2), \qquad \lambda_Z^* = 2 + \mathcal{O}(\epsilon). \qquad (15)
$$

Hence, the fixed point for $Z(\phi)$ appears at order $\mathcal{O}(1)$, and it is not influenced by the explicit form of (14). As a whole, the system has a non-trivial fixed point in the space of couplings, namely $(G^*, \lambda^*\phi, \lambda_V^*\phi, \lambda_Z^*\phi)$, which turns out to be a saddle point. In fact, when studying the stability of the fixed point (11), (14), (15), we can perform variations along four different trajectories. A careful analysis of the beta-functions (13) for the linear Ansatz (12) shows that the two last equations (i.e. those for V and Z) do not produce a new multiplicity of solutions. In other words, for each value of G^* and λ^* we just have one single value of λ_V^* and one of λ_Z^*, that complete the four coordinates of the fixed point. For λ^* we obtain three distinct solutions: the real one (14) and two purely imaginary ones, of order $\epsilon^{-1/2}$, namely

$$\lambda_\pm^* = \pm 2i\sqrt{\frac{24-n}{3n\epsilon}} + \mathcal{O}(\epsilon^0), \tag{16}$$

which correspond to highly oscillating dilaton couplings. Then

$$\lambda_V^* = \frac{24-n}{12\epsilon} + \mathcal{O}(\epsilon^0), \qquad \lambda_Z^* = \frac{48-n}{12} + \mathcal{O}(\epsilon). \tag{17}$$

When $\epsilon \to 0$ all dilatonic couplings are divergent, except for λ_Z^*, which is finite as in the previous case. We see that the explicit fixed point for the dilatonic coupling λ_Z^* appears always at order $\mathcal{O}(\epsilon^0)$, what is quite a distinguished behavior from that of λ_V^* and λ^*. The fixed point (16) is difficult to interprete as a physically acceptable solution. Perhaps it indicates the possibility of some kind of signature-changing transition (a transition to a dilatonic model with a different signature). It would be also interesting to study simulations of such a model (for an introduction, see [24]) near its fixed point.

Expanding the beta functions near the (only real) fixed point, in the way

$$G = G^* + \delta G, \quad \Phi = \lambda^*\phi + \delta\Phi, \quad V = \lambda_V^*\phi + \delta V, \quad Z = \lambda_Z^*\phi + \delta Z, \tag{18}$$

and assuming all the fluctuations to be small, we obtain

$$\begin{aligned}
\delta\beta_G &= -\epsilon\delta G, \\
\delta\beta_\Phi &= \frac{\epsilon}{2}\left(e^{2\phi}-1\right)\frac{d}{d\phi}\delta\Phi + \mathcal{O}(\epsilon^2), \\
\delta\beta_V &= \frac{\epsilon}{2}\left(\frac{12-n}{24-n}e^{2\phi}-1\right)\frac{d}{d\phi}\delta V + \mathcal{O}(\epsilon^2), \\
\delta\beta_Z &= \frac{1}{2}\left(e^{2\phi}-1\right)\frac{d}{d\phi}\delta Z + \mathcal{O}(\epsilon).
\end{aligned} \tag{19}$$

As observed in ref. [14], we may take e^ϕ to play the role of loop expansion parameter, and restrict ourselves to the region $e^{2\phi} \le 1$, that is $-\infty < \phi \le 0$. The change of variables

$$\eta_1 = \ln(e^{-2\phi}-1), \qquad \eta_2 = \ln\left[\frac{30-n}{12}\left(e^{-2\phi} - \frac{18-n}{30-n}\right)\right], \tag{20}$$

transform this region into the following ones

$$
\begin{array}{c}
\phi = 0, \quad \eta_1 \to -\infty, \quad \eta_2 = 0, \\
\phi \to -\infty, \quad \eta_1 \to +\infty, \quad \eta_2 \to +\infty,
\end{array}
\tag{21}
$$

respectively. They simplify expressions (19), which now read:

$$
\begin{aligned}
\delta\beta_G &= -\epsilon\delta G, \\
\delta\beta_\Phi &= \epsilon\frac{d}{d\eta_1}\delta\Phi + \mathcal{O}(\epsilon^2), \\
\delta\beta_V &= \epsilon\frac{d}{d\eta_2}\delta V + \mathcal{O}(\epsilon^2), \\
\delta\beta_Z &= \frac{d}{d\eta_1}\delta Z + \mathcal{O}(\epsilon).
\end{aligned}
\tag{22}
$$

A similar analysis to the one carried out in [20] shows that the fixed points for V and Z are ultraviolet stable in the direction $\delta\Phi$ but are always infrared unstable in this direction. The point (11), (14), (15), is a saddle fixed point of the RG equations.

Thus we have showed that the general dilatonic gravity theory (1) has an asymptotically free regime for the gravitational coupling constant in which the theory possesses a non-trivial ultraviolet fixed point for all dilatonic couplings.

We shall now see explicitly how the theory (1) can be represented in a very simple form at the non-trivial fixed point (14), (15), displaying an asymptotically free gravitational coupling constant. To this end let us perform a Weyl transformation of the action as follows

$$
g_{\mu\nu} \longrightarrow g_{\mu\nu} \exp\left(\frac{4\lambda^*}{\epsilon}\phi\right). \tag{23}
$$

Then

$$
\begin{aligned}
S = & \int d^dx\,\sqrt{-g}\,\left\{\frac{\mu^\epsilon}{16\pi G^*}e^{-2(1-\lambda^*)\phi}\left[R - \frac{4(1+\epsilon)}{\epsilon}\lambda^*(2-\lambda^*)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right] - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi_i\partial_\nu\chi^i\right. \\
& \left. + \mu^\epsilon m^2 \exp\left[\left(2\lambda^* + \frac{4\lambda^*}{\epsilon} - \lambda_V^*\right)\phi\right] + \frac{\mu^\epsilon}{2}\exp\left[\left(2\lambda^* - 2\lambda_Z\right)\phi\right]g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right\}.
\end{aligned}
\tag{24}
$$

As we see here, the general dilatonic gravity theory we have been considering can be cast, at the non-trivial fixed point and near two dimensions, under the form (24), where the n scalars do not interact explicitly with the dilaton. However, though being of the form of a string-inspired dilatonic model, (24) does not represent the CGHS action [5]. In this respect, it is interesting to notice that the particular case of the model (1) with $\exp[-2Z(\phi)] = 0$ has indeed this feature: it also possesses a non-trivial ultraviolet fixed point at which it can be represented by the CGHS action (see [14, 20]).

The model (24) is finite for $\epsilon \to 0$ (in exactly two dimensions) and one can now study different properties of it, as two-dimensional black hole solutions (see [11, 12]), once we get to two dimensions.

3 Fermion-dilatonic gravity near two dimensions.

We can easily extend the above picture to other forms of matter. As an example we shall here consider the quite interesting case of dilatonic gravity interacting with m Majorana fermions. The corresponding Lagrangian is

$$L = \frac{\mu^\epsilon}{2} e^{-2Z(\phi)} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{\mu^\epsilon}{16\pi G} e^{-2\phi} R - \frac{i}{2} e^{-q(\phi)} \bar{\psi}_a \gamma^\lambda \partial_\lambda \psi_a, \tag{25}$$

where ψ_a is an m-component Majorana spinor. The one-loop renormalization of this theory can be done in close analogy with the case of dilaton-scalar gravity. Using the one-loop counterterms found in Ref. [15], we can write

$$\Gamma_{count} = -\mu^\epsilon \int d^d x \sqrt{-g} \left[R A_1 + g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \bar{A}_2(\phi) + e^{-q(\phi)} \mu^{-\epsilon} A_3(\phi) T \right], \tag{26}$$

where

$$\begin{aligned} T &= -\frac{i}{2} \bar{\psi}_a \gamma^\lambda \partial_\lambda \psi_a, \qquad A_1 = \frac{48-m}{48\pi\epsilon}, \\ \bar{A}_2(\phi) &= A_2 + e^{-2Z(\phi)} A(\phi) = -\frac{2}{\pi\epsilon} - \frac{4G}{\epsilon} e^{2\phi - 2Z(\phi)} [2 - Z'(\phi)], \\ A_3(\phi) &= -\frac{4G}{\epsilon} e^{2\phi} \left[\frac{3}{4} - \frac{q'(\phi)}{2} + 12\pi G e^{2\phi - 2Z(\phi)} \right]. \end{aligned} \tag{27}$$

The beta-functions have the same form as in (8)-(11) (with $A_2(\phi) = A_2$), and instead of β_ϕ and β_V we have just one beta function.

$$\begin{aligned} \beta_q(\phi_0) &= \mu \frac{\partial q(\phi_0)}{\partial \mu} = 8\pi\epsilon A_1 G (e^{2\phi_0} - 1) q'(\phi_0) \\ &+ \frac{\pi\epsilon G}{\epsilon + 1} A_2 (e^{2\phi_0} - 1) [\epsilon q'(\phi_0) - 2(\epsilon + 1)] - \epsilon [A_3(\phi_0) - A_3(0)]. \end{aligned} \tag{28}$$

Now, using the linear Ansatz

$$Z(\phi) = \lambda_Z \phi, \qquad q(\phi) = \lambda \phi, \tag{29}$$

we get the ultraviolet fixed point

$$\begin{aligned} G^* &= \frac{3\epsilon}{48 - m}, \ \epsilon > 0, \ m < 48, \\ \lambda_Z &= 2 + \mathcal{O}(\epsilon), \ m \neq 24 \ (\lambda_Z = 3, \text{for } m = 24), \\ \lambda_q &= \frac{336}{48 + m} + \mathcal{O}(\epsilon). \end{aligned} \tag{30}$$

Hence, we obtain once more a non-trivial fixed point, but it is to be noticed that the behavior of the fermion-dilaton coupling is qualitatively different from the one corresponding to the scalar-dilaton coupling (where $\lambda \equiv \epsilon + \mathcal{O}(\epsilon^2)$). In a similar way, a more complicated theory of dilatonic gravity with both scalar and fermionic matter could be considered.

4 Jackiw-Teitelboim and lineal gravities near two dimensions.

In this section we consider the one-loop renormalization of two popular models of dilatonic gravity, namely the Jackiw-Teitelboim model [25] and lineal gravity [26], near two dimensions. We will study the situation when such models of dilatonic gravity are connected with scalar matter in the same way as in our starting Lagrangian (1). The corresponding Lagrangians are now:

$$L_{JT} = \frac{\mu^\epsilon}{16\pi G} e^{-2\phi} \left(R + \Lambda\right) - \frac{1}{2} e^{-2\Phi(\phi)} g^{\mu\nu} \partial_\mu \chi_i \partial_\nu \chi^i \tag{31}$$

and

$$L_{lg} = \frac{\mu^\epsilon}{16\pi G} \left(e^{-2\phi} R + \Lambda\right) - \frac{1}{2} e^{-2\Phi(\phi)} g^{\mu\nu} \partial_\mu \chi_i \partial_\nu \chi^i. \tag{32}$$

As one can infer from the discussion in the previous section, these two models are not fixed points of the RG in $2+\epsilon$ dimensions. It is therefore natural to consider the one-loop renormalization of these models from the very beginning, in order to see how their specific properties actually influence the renormalization structure. In particular, one might expect that the renormalization of both models should be performed in a quite different way —as compared with the renormalization of the general dilatonic model previously considered.

The calculation of the one-loop effective action is done in the same gauge (2) as before, with the following result:

$$\Gamma^{JT}_{div} = \frac{1}{4\pi\epsilon} \int d^d x \, \sqrt{-g} \left\{ \frac{24-n}{6} R - \left[8 - n\Phi'(\phi)^2\right] g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + 4\Lambda \right\} \tag{33}$$

and

$$\Gamma^{lg}_{div} = \frac{1}{4\pi\epsilon} \int d^d x \, \sqrt{-g} \left\{ \frac{24-n}{6} R - \left[8 - n\Phi'(\phi)^2\right] g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + 2\Lambda e^{2\phi} \right\}, \tag{34}$$

respectively. At first look it would seem that the two theories, (33) and (34), are both non-renormalizable in $2+\epsilon$ dimensions. However, the situation is not so simple as it looks. Indeed, to begin with one can always consider a subclass of the theories under discussion with some specific choices for the dilatonic function $\Phi(\phi)$, and things could depend on this choice. On the other hand, the one-loop effective action is, generally speaking, a gauge dependent quantity (for an introduction to the effective action formalism in QG, see [21]).

To illustrate the situation, let us first consider lineal gravity and impose that $e^{-2\Phi(\phi)} = 1$. In this case, in the dilatonic kinetic term in (34) the ϕ-dependence disappears completely. Moreover, one can also choose a gauge of Landau type with gauge parameter $\alpha = 0$. Then, as it was shown in the last of Refs. [10], the second and the third terms in (34) disappear in this gauge (the first term in (34) is a gauge independent quantity). As a result, we obtain that lineal gravity (32) with scalar matter not interacting with the dilaton explicitly is one-loop multiplicatively renormalizable in a gauge of Landau type in exactly-two dimensions. If $n = 24$ then, in addition, it is one-loop finite in $2+\epsilon$ dimensions.

For the Jackiw-Teitelboim model the situation is somewhat different. With the same choice for $\Phi(\phi)$ as above, we do not know of any gauge —as the one found in Ref. [10]— in which both the second and third terms in Γ_{div}^{JT} disappear simultaneously (what should not mean that it does not exist). In the gauge under consideration one can choose $\Phi(\phi) = \sqrt{8/n}\,\phi$ in order to remove the dilatonic kinetic term. However, the last term in (33) is still present. Hence, it is very likely that, in fact, the Jackiw-Teitelboim model is not one-loop renormalizable, either in exactly-two or in $2+\epsilon$ dimensions.

An important question in relation with two-dimensional dilatonic gravity is the existence of solutions of black hole type (for a review see [11, 12]). It would be of interest to understand something about the structure of black holes in $2+\epsilon$ dimensions. In particular, let us consider the Jackiw-Teitelboim model, where it is known that there exists a regular, asymptotically flat black hole spacetime [27]. Starting from the action (31) (without the scalars χ_i, for simplicity), we expect that the regular black holes near two dimensions will be described by a metric of Schwarzschild type with the dilaton, of the form

$$\begin{aligned} ds^2 &= -\left(\frac{\Lambda}{2}r^2 - M\right)dt^2 + \frac{dr^2}{\frac{\Lambda}{2}r^2 - M} + \left[f(r)d\theta\right]^2\epsilon, \\ \phi &= r + \mathcal{O}(\epsilon), \end{aligned} \tag{35}$$

where $f(r)$ is some regular function of the radius. Of course, a rigorous mathematical description of the classical theory for non-integer ϵ is lacking.

Another issue connected with the two models above is the possibility of a gauge formulation of the same, using an extended Poincaré group theory (see for example [26]). It would be again of interest to study such questions in the $(2+\epsilon)$-dimensional formalism.

5 Discussion.

In this paper we have studied a general theory of dilatonic gravity with n scalars, near two dimensions, at the quantum level. The one-loop β-functions have been calculated and a non-trivial ultraviolet fixed point of the theory has been found. At the fixed point the gravitational coupling constant of the theory is asymptotically free, and the theory can be given a form in which the scalars do not interact with the dilaton. By performing convenient transformations, one can also give the action some other, different forms at the fixed point. For instance, the transformation

$$g_{\mu\nu} \longrightarrow g_{\mu\nu}\exp\left(\frac{2\lambda_V}{2+\epsilon}\phi\right), \tag{36}$$

converts the theory (1) with the dilatonic potential $m^2 e^{-\lambda_V^*\phi}$ into a theory with cosmological constant m^2 (no dilatonic potential). Fermion-dilatonic gravity near two dimensions has been considered too.

A further remark is connected with the possibility to increase the number of scalars (maintaining always the regime of asymptotic freedom) by adding to the theory a Yang-Mills action with a dilatonic coupling [20]

$$L_{YM} = \frac{1}{4} e^{-f_2(\phi)} (G^a_{\mu\nu})^2, \qquad a = 1, 2, \ldots, N. \tag{37}$$

Then, only A_1 is going to change in Eqs. (6) and, as a result, one can show that the following ultraviolet stable fixed point appears

$$\begin{aligned} G^* &= \frac{3\epsilon}{2(24+6N-n)}, \qquad \lambda^* = -\frac{6\epsilon}{24+6N-n}, \\ \lambda_V^* &= \frac{12\epsilon}{12+6N-n}, \qquad \lambda_f^* = -\frac{36\epsilon}{12+6N-n}, \end{aligned} \tag{38}$$

where we have chosen $f_2(\phi) = \lambda_f \phi$, and λ_Z^* does not change.

Hence, by increasing the dimension N of the gauge group we may increase the number of scalars in the theory, while keeping always the condition that we are in the situation with G asymptotically free. The presence of the first term in (1) does not modify this conclusion.

As a final remark let us mention that the $(2+\epsilon)$-dimensional formalism has been shown [28] to be very useful in understanding the gravitational dressing of the RG beta-function [29] in the case of the sigma model interacting with two-dimensional Einstein gravity. It is a challenge to understand the gravitational dressing of the RG in the sigma model with dilatonic gravity. The results of our discussion here are expected to be quite helpful in this respect.

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